ISSN: 2581-5989
PubMed - National Library of Medicine - ID: 101738774

International Journal of Dental Science and Innovative Research (IJDSIR)

IJDSIR : Dental Publication Service

Available Online at: www.ijdsir.com

Volume – 2, Issue – 3, May - June- 2019, Page No. : 555 - 561

Arch Perimeter Calculation Using Ellipse Equation

Neeraj Saini[1], Sridhar Kannan[2], Nitin Arora[3], Ashish Kumar Singh[4], Abhita Malhotra[5],Sajna Fassul[6]

[1]Post graduate, Department of Orthodontics, Manav Rachna Dental College, Faridabad- 121004

[2]Professor and Head of Department, Department of Orthodontics, Manav Rachna Dental College, Faridabad- 121004

[3]Professor- Department of Orthodontics, Manav Rachna Dental College, Faridabad- 121004

[4]Reader- Department of Orthodontics, Manav Rachna Dental College, Faridabad- 121004

[5]Senior Lecturer - Department of Orthodontics, Manav Rachna Dental College, Faridabad- 121004

[6]Post graduate- Department of Orthodontics, Manav Rachna Dental College, Faridabad- 121004

Corresponding Author: Sridhar Kannan**,** Professor and Head of Department, Department of Orthodontics, Manav Rachna Dental College, Faridabad- 121004

Type of Publication: Original Research Paper

Conflicts of Interest: Nil

Abstract

Objective: To find mathematical correlation of measured arch perimeter with calculated arch perimeter using equation for ellipse given by Ramanujan's and by using Ramanujan's equation arch perimeter prediction by transverse expansion of posterior segments and anteriors proclination.

Materials and Methods: The sample comprises of 120 study models of untreated patients in the age range of 18-25 years. Measurement of maxillary and mandibular arch perimeter by using occlusogram in 1:1 magnification (photocopying method).Calculated perimeter was obtained by applying the equation of Ramanujan'sfor ellipse and correlated with the perimeter value obtained from measurement. Measurements directly on midbuccal surfaces of maxillary and mandibular arches linearly and circumferentially.

Results: Results show a high correlation of 0.965 and 0.990 in maxillary and mandibular arch respectively by using Pearson correlation test at 0.01 levels and an error of 1.7% and 2.42%. Results obtained were put in equation of ellipse by Ramanujan's for arch perimeter prediction by transverse expansion of posterior segments and anteriors proclination.

Conclusions: Ramanujan's ellipse equation can be used for calculation of arch peri meter effectively. Prediction of arch perimeter gains by proclining the incisors and intermolar expansion in maxillary and mandibular arches can also be done.

Keywords: Ramanujan's equation, Arch perimeter

Introduction

A major tool in planning orthodontic treatment is model analysis which helps in assessing the discrepancy between tooth material and arch length. There should be harmonious relationship between the two, if tooth material is more than the arch length the space available for alignment is not sufficient resulting in crowding and if tooth material is less than the space available then there will be spacing. Analysis of arch perimeter is the best method to predict about the space available or space required for alignment of the teeth. Following the arch perimeter strictly would result in functional, esthetic, and

stable orthodontic treatment results[1]. Therefore an accurate method of determining the arch length is essential in orthodontics.

Many researchers seek for universal arch form till now, but due to nature's law nothing is consistent and every human upper and lower arches are varying makes the geometric comparison almost inevitable. The shape of dental arches could be described as different geometric curves such as the ellipse[2], conic sections[3]and beta function[4].

According to Stanton[5] and Begole[6], for a dental arch curvature to be explained till now is mathematical curve and that should have potential of varying shapes according to human dental arch.

Currier[2]proposed that the ellipse till now is the most appropriate geometrical figure for representing both upper and lower arches. This geometrical figure permits stable representation of dental arch. By varying the values of variables prediction of arch perimeter can be done which requires less effort. There are several aforementioned studies in the literature for the calculation of arch perimeter by using different formulas Beta function[4], Polynomial function [7], Fourier series[8], these formulas need deep knowledge of mathematics for calculation/prediction of arch perimeter by transverse expansion of posterior segments and anteriors proclination because of their intricacies, many variables and time consuming calculation they cannot be used in regular clinical practice.

Srinivasan Ramanujan[9] formulated an equation for perimeter calculation of ellipse by only using two variables that is ["a" semimaj or axis and "b" semi in or axis], among the entire complex and in tricated formulas in the literature the Ramanujan's equation of ellipse is easy and applicable in clinical practice because of only two variables. In the present study Ramanujan's equation of ellipse is used for calculation of arch perimeter and also predicting arch perimeter gained by proclining the incisors and intermolar expansion in maxillary and mandibular arches.

The primary objective of this study was to measure the maxillary and mandibular arch perimeter by using occluso gram in 1:1 magnification and perimeter calculation of upper and lower arches by applying equation for ellipse given by Ramanujan's and secondary objective was by using the ellipse equation given by Ramanujan's arch perimeter prediction by transverse expansion of posterior segments and anteriors proclination.

Materials and Method

The sample comprised of 120 study models of upper and lower arch who had visited the Department of Orthodontics in ManavRachna Dental College Faridabad, on patients who met the inclusion criteria and the prior explanation of the procedure with their informed consent. All the patients between 18-25 years of age, well aligned dental arches, full set of dentition from second molar to second molar and no prior orthodontic treatment were included in the study. The patients having crowding more than 2mm, tooth shape and size anomaly ex- Peg laterals and missing teeth were excluded.

Arch perimeter measurement of upper and lower dental arches

For determination of measured perimeter, the Yen[10] technique of photocopying and scanning the upper and lower casts was used and then measurement of perimeter was done. Arch perimeter was determined from the distobuccal cusp of the right maxillary first molar todistobuccal cusp of contra lateral first molar using occlusogram and brass wire. Markings were made on the brass wire at the level of distobuccal cusp, the wire was then straightened and laid flat on a grid paper and the markings transferred to the grid paper, which was then

further evaluated by using a digital caliper(12inch-300mm Electronic vernier caliper).

Calculation of arch perimeter of Upper and Lower arch using Ramanujan's equation- Semiminor axis 'b' for Upper and Lower arch-

Semi minor axis was estimated between the distobuccal cusps of first molars of both sides from the midbuccal point. This point was marked with a pencil for further measurements.

The value by this measurement is complete minor axis that should be divided by half to get the actual value of semi minor axis (b).

Semi minor axis for maxillary arch (figure 1(i)). Semi minor axis for mandibular arch (figure 1(ii)).

Semi major axis 'a' for maxillary and mandibular arch

Distance from the line connecting the central incisors to the semi minor axis was measured with a digital calliper to estimate Semi major axis Semi major axis for Upper arch [Figure 1(i)]. Semi major axis for Lower arch [Figure 1(ii)] The value of 'a' and 'b' were then inserted into the Ramanujan's equation to obtain the perimeter.

Perimeter of an ellipse = $\pi(a+b)\ \{1+3h/(10+\sqrt{(4-3h)}\}$ where $h= (a-b)^2/ (a+b)^2$

To reduce errors, all measurements were done by a single investigator, both the maxillary and mandibular casts were re measured three times in one week's interval and their mean was taken.. All models were measured using digital caliper to an accuracy of 0.01 mm and the reliability was evaluated using the Dahlberg's formula[14].

Applying Pearson correlation test on the value that was obtained by using ellipse equation given by Ramanujan's and measured perimeter on scanned models of upper and lower arch.

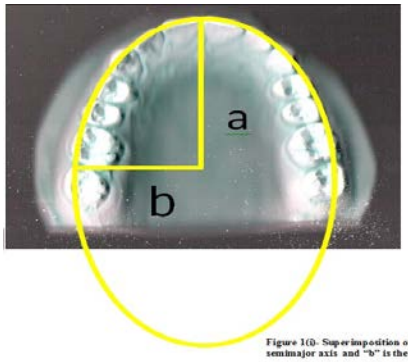


Figure 1(i)- Superimposition of ellipse on maxillary arch with "a" representing the semimajor axis and "b" is the semiminor axis.

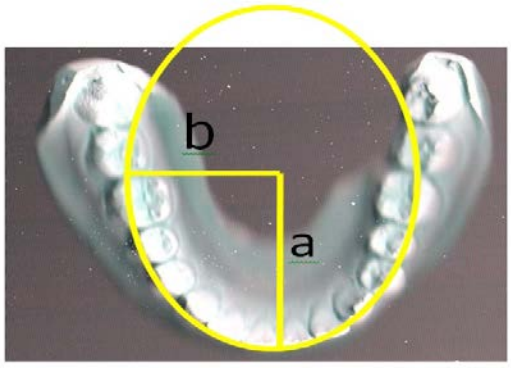
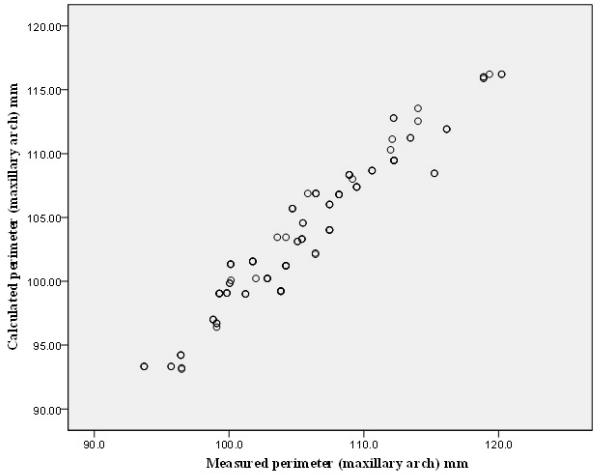


Figure 1(ii)- Superimposition of ellipse on mandibular arch with "a" representing the semimajor axis and "b" is the semiminor axis.

Results

Results show a high correlation of 0.965 and 0.990 in maxillary and mandibular arch respectively by using Pearson correlation test at 0.01 level and an error of 1.7% and 2.42%.(Graph-1)(Table-1).



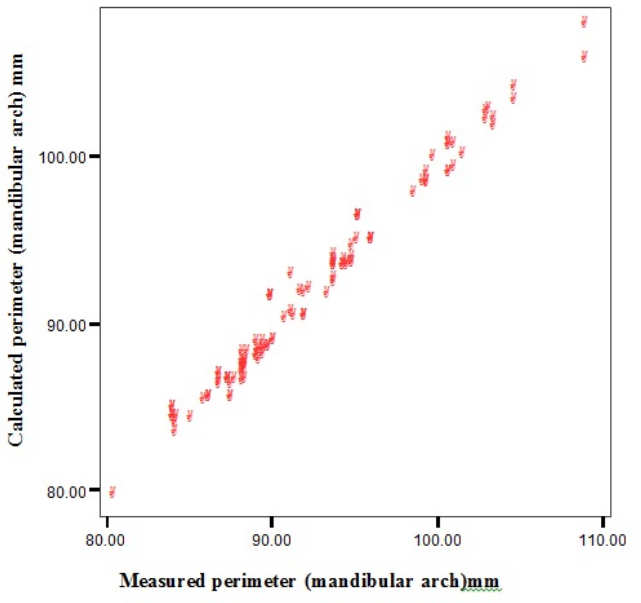
Graph 1- Correlation between measured and calculated arch perimeter of maxillary and mandibular arch

Table 1: Correlation between calculated and measured perimeter

	Pearson correlation	Significance*	Mean error	Range
Maxillary arch	0.965	0.01	1.79	1.2-6.78
Mandibular arch	0.990	0.01	2.42	1.19 – 3.61

*Correlation is significant at 0.01 level

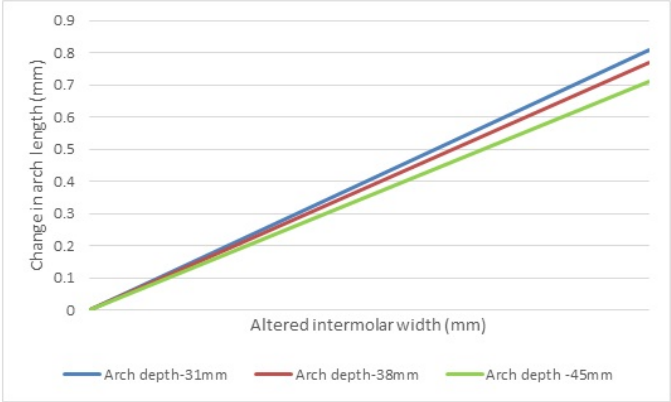
Transverse changes can be plotted by fixing the arch length for maxillary arch. At 31mm arch length there is 0.81mm increase in perimeter with 1mm of expansion. At 38mm arch length, 1mm expansion yields only 0.77mm perimeter gain and at 40mm the value is 0.71mm further decreases (Table-2). Variations in arch length gave a clinically insignificant change in perimeter value within 0.10mm.Incisor proclination can be plotted by fixing the transverse dimension fixed for maxillary arch. At 26mm transverse dimension there is 1.71mm increase in perimeter with 1mm of incisor proclination. At 35 mm transverse dimension 1mm incisor proclination yields only 1.66mm perimeter gain and at 44mm the value is 1.61mm further decreases (Table-3). Variations in arch length gave a clinically insignificant change in perimeter value within 0.10mm.(Graph-2)



Graph 2- Arch perimeter change when modifying the transverse width at fixed arch depth and altering arch depth when transverse width is fixed in maxillary arch

Table 2: Change in maxillary arch perimeter when altering the intermolar width at fixed arch depth

Arch depth fixed at	Change in arch perimeter per mm change in intermolar width	Range
31 mm	0.81	0.10
38 mm	0.77	
40 mm	0.71	



Graph 3- Arch perimeter change when modifying the transverse width at fixed arch depth and altering arch depth when transverse width is fixed in mandibular arch

Table 3: Change in maxillary arch perimeter when altering the arch depth at fixed intermolar width

Intermolar width fixed at	Change in arch perimeter per mm change in intermolar width	Range
26 mm	1.71	0.10
35 mm	1.66	
44 mm	1.61	

Transverse changes can be plotted by fixing the arch length for mandibular arch. At 27mm arch length there is 1.08mm increase in perimeter with 1mm of expansion. At 33mm arch length, 1mm expansion yields only 0.97mm perimeter gain and at 40mm the value is 0.96mm further decreases (Table-4).Variations in arch length gave a clinically insignificant change in perimeter value within 0.12mm. Incisor proclination can be plotted by fixing the transverse dimension fixed for maxillary arch. At 24mm transverse dimension there is 0.83mm increase in perimeter with 1mm of incisor proclination. At 27 mm transverse dimension 1mm incisor proclination yields only 0.76mm perimeter gain and at 31mm the value is 0.74mm further decreases (Table-5). Variations in arch length gave a clinically insignificant change in perimeter value within 0.09mm. (Graph-3)

Table 4: Change in mandibular arch perimeter when altering the intermolar width at fixed arch depth

Arch depth fixed at	Change in arch perimeter per mm change in intermolar width	Range (mm)
27 mm	1.08	0.12
33 mm	0.97	
40 mm	0.96	

Table 5: Change in mandibular arch perimeter when altering the arch depth at fixed intermolar width

Intermolar width fixed at	Change in arch perimeter per mm change in intermolar width	Range
24 mm	0.83	0.09
27 mm	0.76	
31 mm	0.74	

Discussion

Finding the universal arch form has been an enigma and a challenge for orthodontists. The problem which was persisting from beginning and mentioned by Stanton[5] and Begole[6]for a dental arch curvature to be explained till now is mathematical curve and that should have potential of varying shapes according to human dental arch.

In this study ellipse was chosen due to a unique property of this conic section, its eccentricity 0<e<1, and a wide variety of shapes can be approximated with this conic section.

Srinivasn Ramanujan's[9], gave an equation for calculation of arch perimeter of an ellipse, which is the most easy and clinically applicable formula till now. When using Ramanujan's formula for maxillary arch, for every millimeter of transverse expansion 0.81mm gain of arch perimeter was seen and average of 1.61mm increase of arch perimeter was noted for 1millimeter of anterior proclination. For mandibular arch for every 1 millimeter of transverse expansion 1.08mm gain of arch perimeter was seen and average of 0.83mm increase in arch perimeter was noted for 1millimeter of anterior proclination.

Our study findings have high correlation with that of Adkins et al[12], which concluded a perimeter gain of 4.7mm by 6.5mm expansion which is an average of 0.72mm for 1mm of transverse expansion.

Germane et al[13] reported arch perimeter increase of 0.27 with 1mm of arch expansion and arch perimeter increase of 1.04 mm for 1 mm of incisor proclination which was more than that with arch expansion. This correlated well with our study. In this study inter canine expansion was also done which showed arch perimeter gain of - 0.73mm per millimeter of expansion[17]. Increasing arch perimeter by incisor proclination was seen to be 4 times effective compared to perimeter gain from expansion. Whereas perimeter gain from proclination was seen to be only twice effective as compared to expansion from our study.

The study done by Paulino et al[14] determined the correlation between the arch length, inter canine width and intermolar width and predicted some of these measurements based on other. Their finding says that very high correlation exist between Intercanine width and arch length, both for upper and lower arches in males and females. They concluded that for an increase of 1mm inters canine width, the arch length increases approximately 1.36mm. Though Ramanujan's ellipse equation don't give perimeter changes from inter canine expansion, indirect calculations can be done using computer software.

Ricketts et al[15] suggested that an increase in arch perimeter of 0.25mm, a 1mm increase of the intermolar distance, inter canine distance and 1mm of arch length produces arch perimeter increase of 0.25mm,1mm and 2mm respectively whereas the perimeter gain was 2 times more that is 0.81mm with 1mm intermolar expansion in our study.

Mutinelli et al[16]noticed an increase in arch perimeter of 1.51mm by 1mm proclination of lower incisors while inter canine distance was kept constant showing less correlation with present study.

Strong Points of Study

- The present study is extensive as it is using 120 study models of both maxillary and mandibular arch of class I cases with minimal crowding.
- Prediction of arch perimeter changes in both ideal symmetrical and in skewed dental arches.
- In unilateral constricted arches, the normal side semi minor axis could be substituted in the equation and the resulting value could be compared with the original perimeter to determine the efficiency of expansion of the constricted side for reduction of crowding.
- Possibility of crowding resolution can be evaluated in subdivision cases with asymmetric anteroposterior position of molars on either side by substitution of values of '*a*' and'*b*' according to the right or the left molar.

Limitation of the Study

Semi minor axis was measured in the distobuccal region of first molar is the major limitation of the study. Arch form (especially tapered) would fit more precisely in the ellipse equation if minor axis was measured distally in the second molar region.

Future Scope

- The future studies should focus on correlation between arch perimeter and expansion in the maxillary and mandibular premolar and second molar regions.
- Study should be further carried out on various types of malocclusion, to find out the differences with respect to age, sex and race and also to predict pretreatment and post treatment changes after orthodontic treatment.

So this research will provide clinicians with a simple way to predict arch perimeter gain during treatment to resolve the arch length tooth material discrepancy.

Clinical Significance

The decision to extract or expand in borderline cases can be simplified by the use of Ramanujan's equation to predict the space gain from analysis of pre-treatment models.

Prediction of arch perimeter gain from proclination can also be predicted accurately by the substitution of the values in the Ramanujan's equation.

The Ramanujan's equation is applicable in a wide variety of clinical situations requiring arch perimeter prediction ranging from simple well aligned arches, crowding cases, and even in cases with skewed arch forms.

The equation is also well adapted for use in situations of Class II subdivision patterns with asymmetrical molars

Conclusion

Amount of arch perimeter gained was 0.81mm, when 1mm of transverse expansion and 1.71mm, when 1mm of anterior proclination in maxillary arch.

Amount of arch perimeter gained was 1.08mm, when 1mm of transverse expansion and 0.83mm, when 1mm of anterior proclination mandibular arch.

Figure Legends

Figure 1(i) - Superimposition of ellipse on maxillary arch with "a" representing the semi major axis and "b" is the semi minor axis.

Figure 1(ii)-Superimposition of ellipse on mandibular arch with "a" representing the semi major axis and "b" is the semi minor axis.

Graph 1- Correlation between measured and calculated arch perimeter of maxillary and mandibular arch

Graph 2- Arch perimeter change when modifying the transverse width at fixed arch depth and altering arch depth when transverse width is fixed in maxillary arch

Graph 3- Arch perimeter change when modifying the transverse width at fixed arch depth and altering arch depth when transverse width is fixed in mandibular arch

Table 1: Correlation between calculated and measured perimeter

Table 2: Change in maxillary arch perimeter when altering the intermolar width at fixed arch depth

Table 3: Change in maxillary arch perimeter when altering the arch depth at fixed intermolar width

Table 4: Change in mandibular arch perimeter when altering the intermolar width at fixed arch depth

Table 5: Change in mandibular arch perimeter when altering the arch depth at fixed intermolar width

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